

# PHYSICS 2DL – SPRING 2010

## MODERN PHYSICS LABORATORY

Monday May 10 2010

Course Week 7

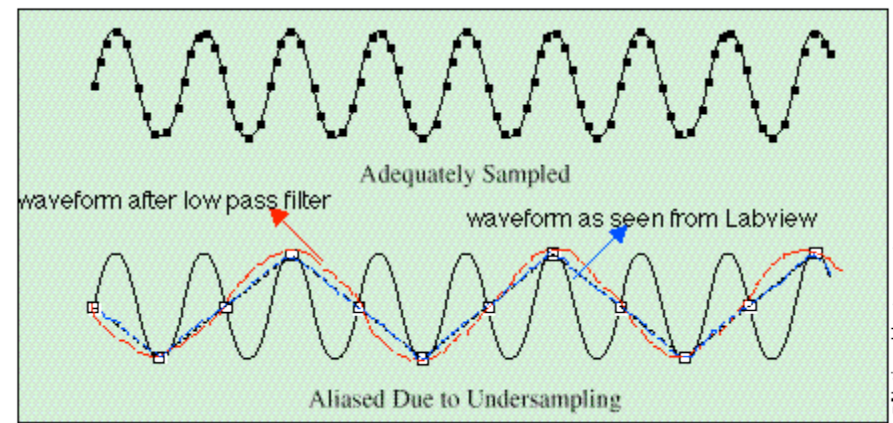
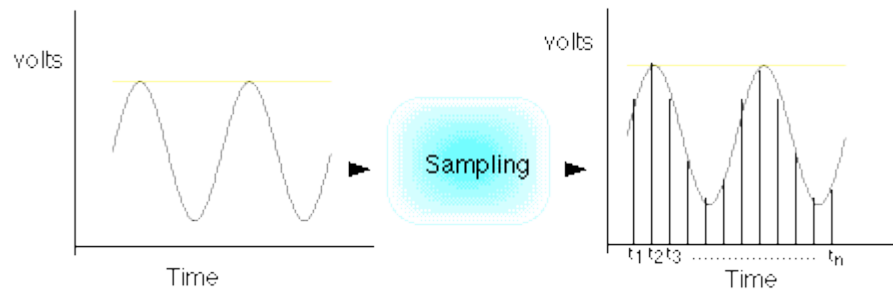
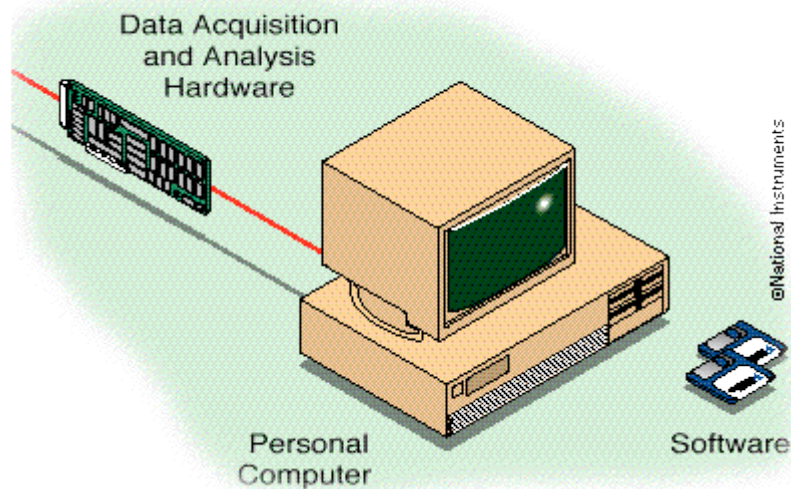
(3 LABs Left!)

Prof. Brian Keating

# 2Day in 2DL

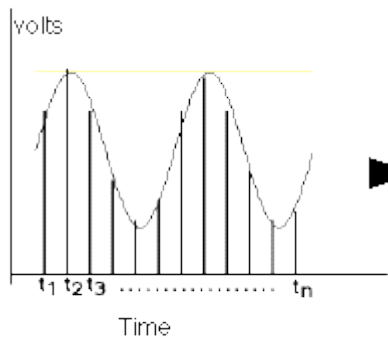
- Questions/Announcements
- Error propagation, chi sq review (ch 8 for last time, ch 10 binomial, ch 11 poisson)
- HW due in lab this week
  - Special Topics: DAQ part 2
  - More on physical constants

# Electronic Measurement using Digital to Analog Conversion



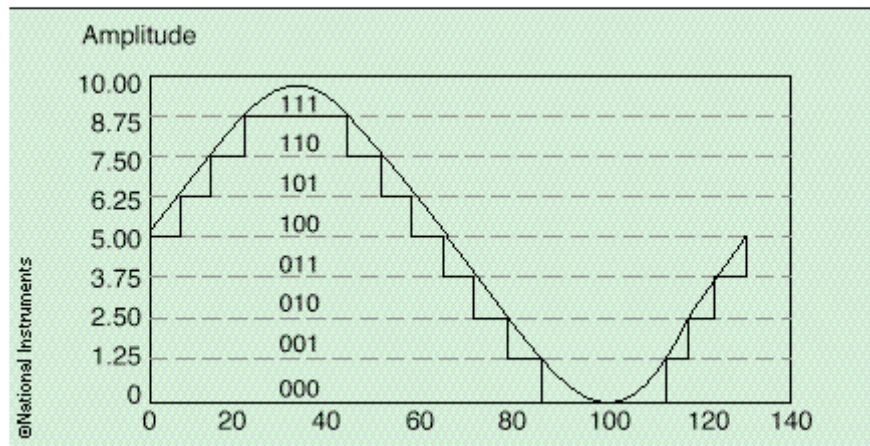
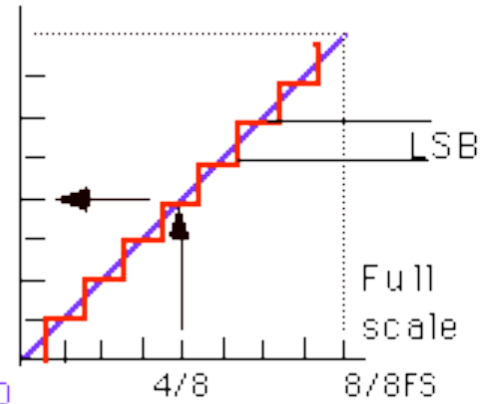
@National Instruments

@National Instruments

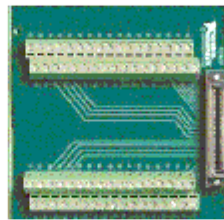


TIME SAMPLE	DIG CODE
$t_1$	110
$t_2$	111
$t_3$	100
...	...
$t_n$	101

digital  
output  
111  
110  
101  
100  
011  
010  
001  
000  
b2b1b0

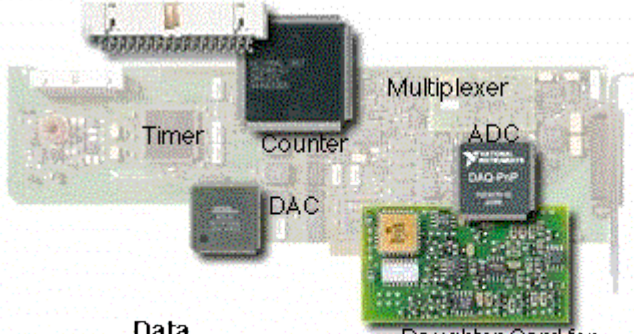


*Outside of the computer*



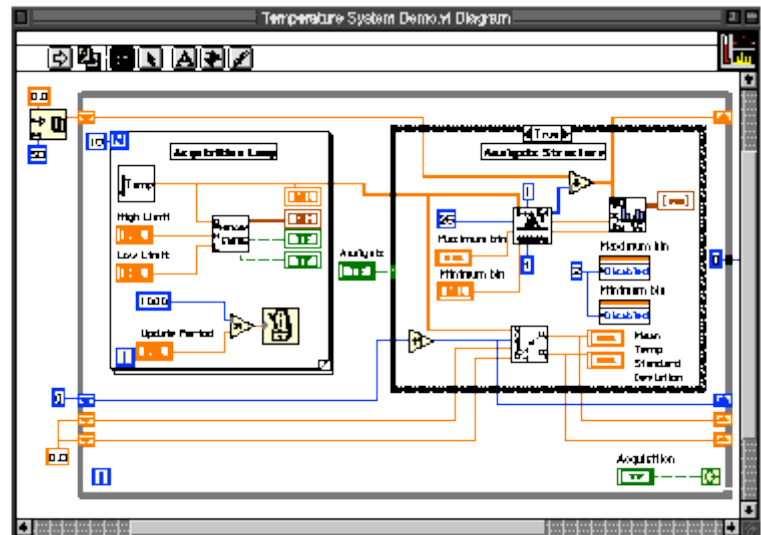
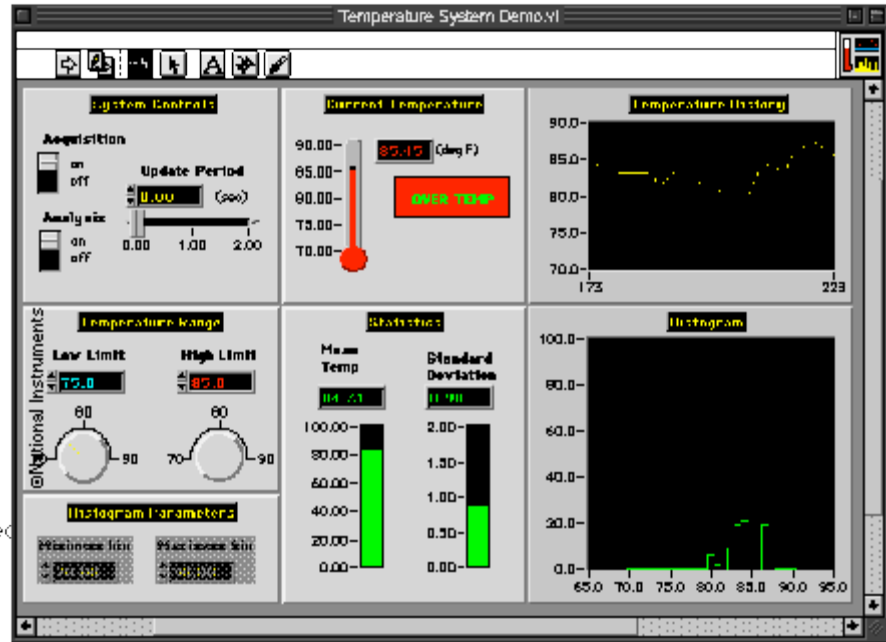
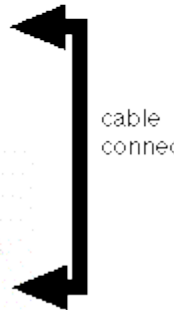
Breakout box

Local Bus connection



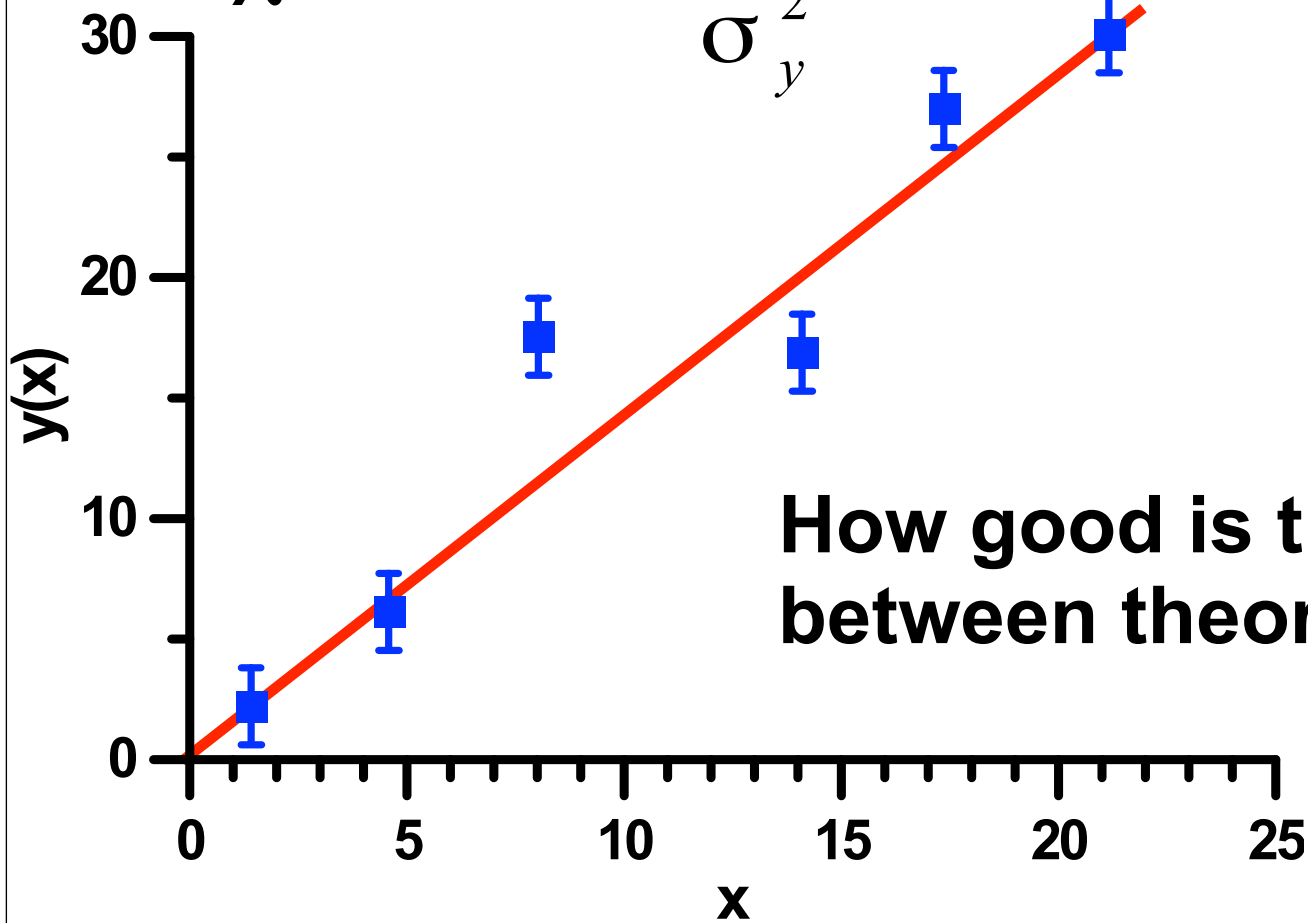
Data Acquisition board

Daughter Card for expansion



# $\chi^2$ TEST for FIT (Ch.12)

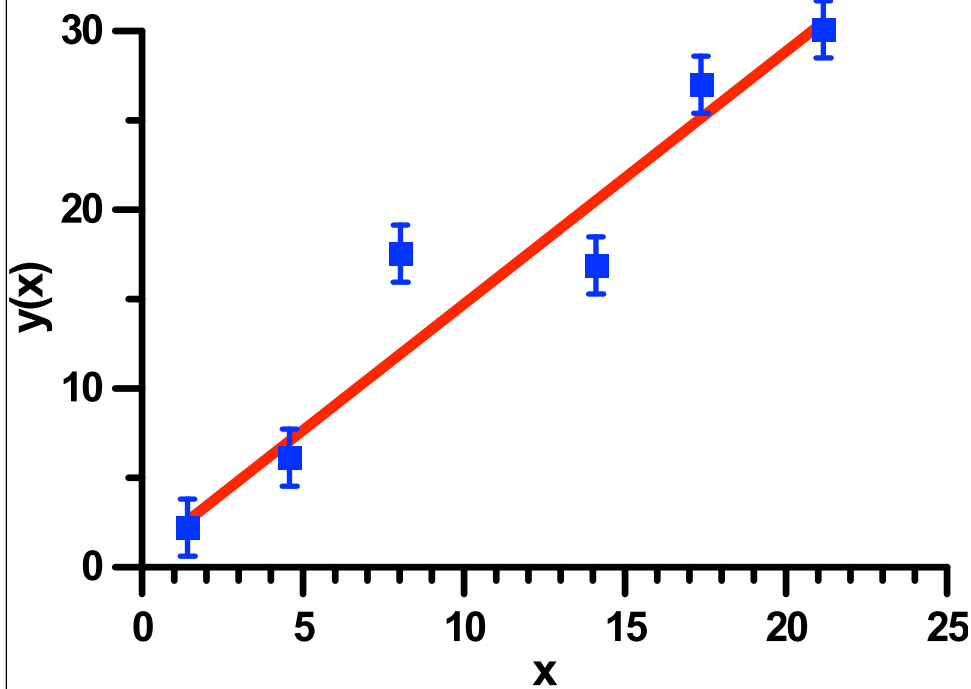
$$\chi^2 = \frac{\sum_{j=1}^N (y_j - f(x_j))^2}{\sigma_y^2}$$



# $\chi^2$ TEST for FIT (Ch.12)

$$\chi^2 = \frac{\sum_{j=1}^N (y_j - f(x_j))^2}{\sigma_y^2} \cong \frac{N\sigma_y^2}{\sigma_y^2} = N$$

$$\tilde{\chi}^2 = \frac{\chi^2}{d} \cong 1$$



**# of degrees of freedom**

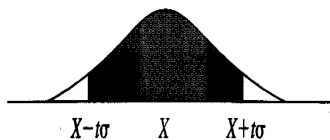
$$d = N - c$$

**# of data points**

**# of parameters calculated from data**

**# of constraints**

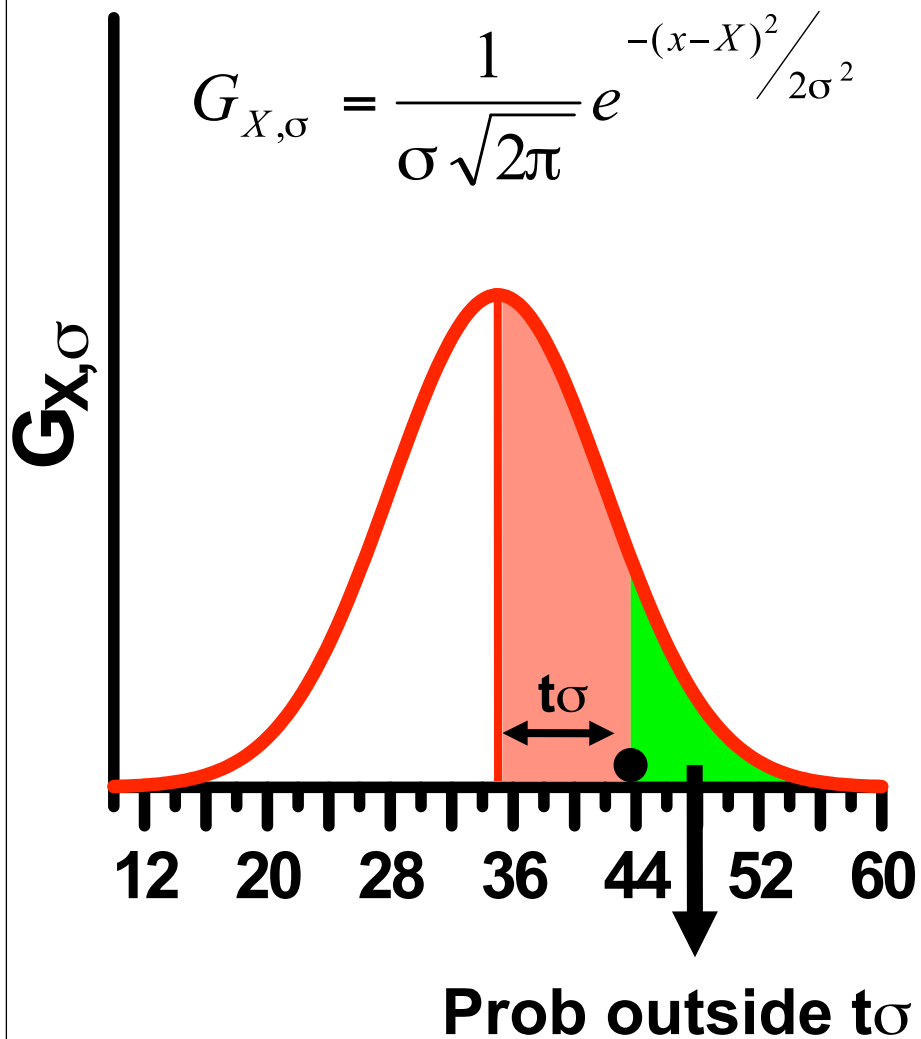
Table A. The percentage probability,  
 $Prob(\text{within } t\sigma) = \int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x) dx,$   
 as a function of  $t$ .



# $\chi^2$ TEST for FIT

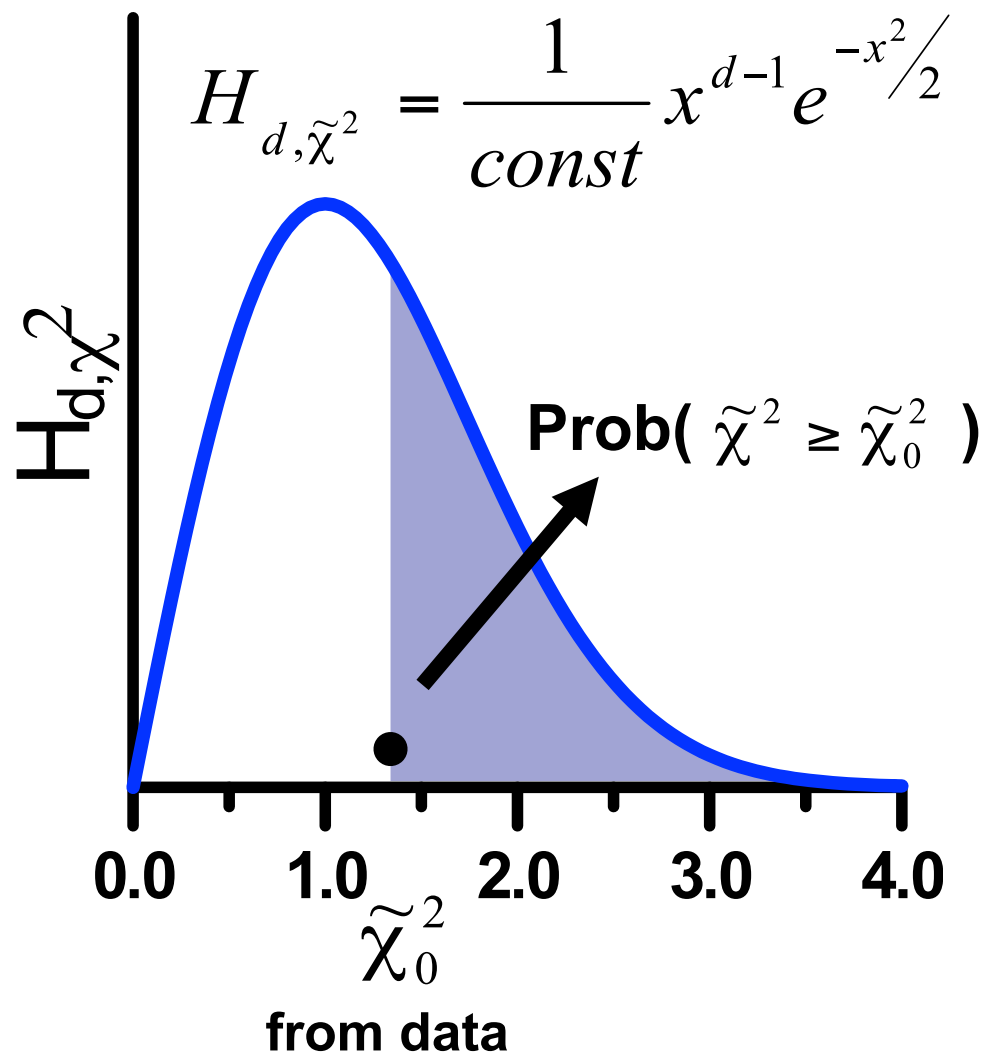
**Gauss distribution:**

$$G_{X,\sigma} = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-X)^2}{2\sigma^2}}$$



**$\tilde{\chi}^2$  distribution:**

$$H_{d,\tilde{\chi}^2} = \frac{1}{const} x^{d-1} e^{-x^2/2}$$





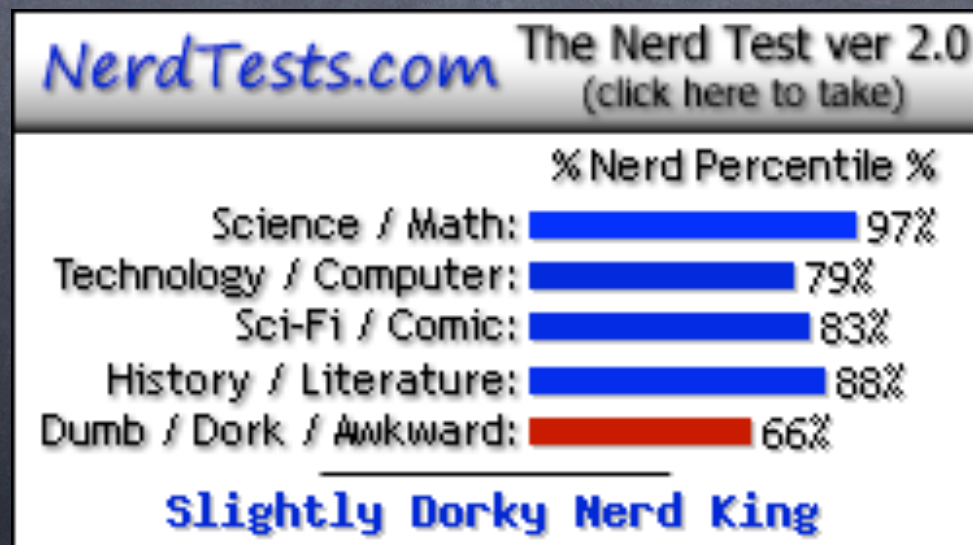
**Table D.** The percentage probability  $Prob_d(\tilde{\chi}^2 \geq \tilde{\chi}_o^2)$  of obtaining a value of  $\tilde{\chi}^2 \geq \tilde{\chi}_o^2$  in an experiment with  $d$  degrees of freedom, as a function of  $d$  and  $\tilde{\chi}_o^2$ . (Blanks indicate probabilities less than 0.05%.)

# Table D

$d$	$\tilde{\chi}_o^2$															
	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	8.0	10.0	
1	100	48	32	22	16	11	8.3	6.1	4.6	3.4	2.5	1.9	1.4	0.5	0.2	
2	100	61	37	22	14	8.2	5.0	3.0	1.8	1.1	0.7	0.4	0.2			
3	100	68	39	21	11	5.8	2.9	1.5	0.7	0.4	0.2	0.1				
4	100	74	41	20	9.2	4.0	1.7	0.7	0.3	0.1	0.1					
5	100	78	42	19	7.5	2.9	1.0	0.4	0.1							
	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
1	100	65	53	44	37	32	27	24	21	18	16	14	12	11	9.4	8.3
2	100	82	67	55	45	37	30	25	20	17	14	11	9.1	7.4	6.1	5.0
3	100	90	75	61	49	39	31	24	19	14	11	8.6	6.6	5.0	3.8	2.9
4	100	94	81	66	52	41	31	23	17	13	9.2	6.6	4.8	3.4	2.4	1.7
5	100	96	85	70	55	42	31	22	16	11	7.5	5.1	3.5	2.3	1.6	1.0
6	100	98	88	73	57	42	30	21	14	9.5	6.2	4.0	2.5	1.6	1.0	0.6
7	100	99	90	76	59	43	30	20	13	8.2	5.1	3.1	1.9	1.1	0.7	0.4
8	100	99	92	78	60	43	29	19	12	7.2	4.2	2.4	1.4	0.8	0.4	0.2
9	100	99	94	80	62	44	29	18	11	6.3	3.5	1.9	1.0	0.5	0.3	0.1
10	100	100	95	82	63	44	29	17	10	5.5	2.9	1.5	0.8	0.4	0.2	0.1
11	100	100	96	83	64	44	28	16	9.1	4.8	2.4	1.2	0.6	0.3	0.1	0.1
12	100	100	96	84	65	45	28	16	8.4	4.2	2.0	0.9	0.4	0.2	0.1	
13	100	100	97	86	66	45	27	15	7.7	3.7	1.7	0.7	0.3	0.1	0.1	
14	100	100	98	87	67	45	27	14	7.1	3.3	1.4	0.6	0.2	0.1		
15	100	100	98	88	68	45	26	14	6.5	2.9	1.2	0.5	0.2	0.1		
16	100	100	98	89	69	45	26	13	6.0	2.5	1.0	0.4	0.1			
17	100	100	99	90	70	45	25	12	5.5	2.2	0.8	0.3	0.1			
18	100	100	99	90	70	46	25	12	5.1	2.0	0.7	0.2	0.1			
19	100	100	99	91	71	46	25	11	4.7	1.7	0.6	0.2	0.1			
20	100	100	99	92	72	46	24	11	4.3	1.5	0.5	0.1				

# Today Ch 10 and Ch 11

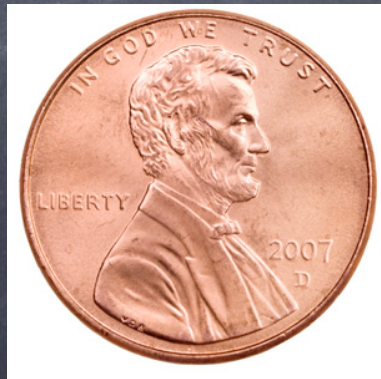
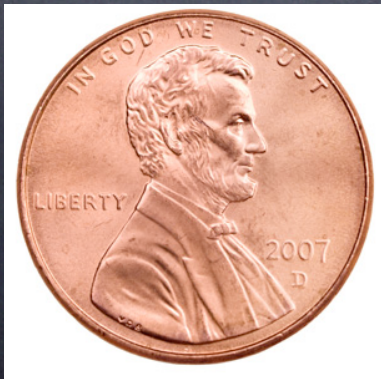
- Review ch 8 least sq fit
  - Ch 10 = Binomial Dist.
- Ch 11 = Poisson



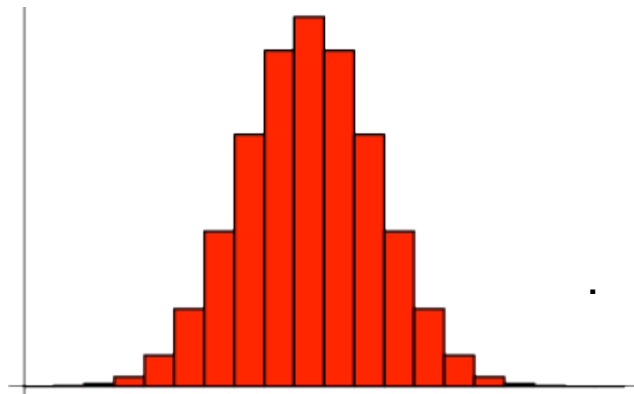
# Ch 10 Binomial Distribution

Why Binomial? Because only 2 outcomes of a given test. Either X happened or it didn't, where 'X' can be a complicated statement like:

“When throwing 3 coins sequentially, what's the probability that the sequence observed was HHT”



# Ch 10 Binomial Distribution



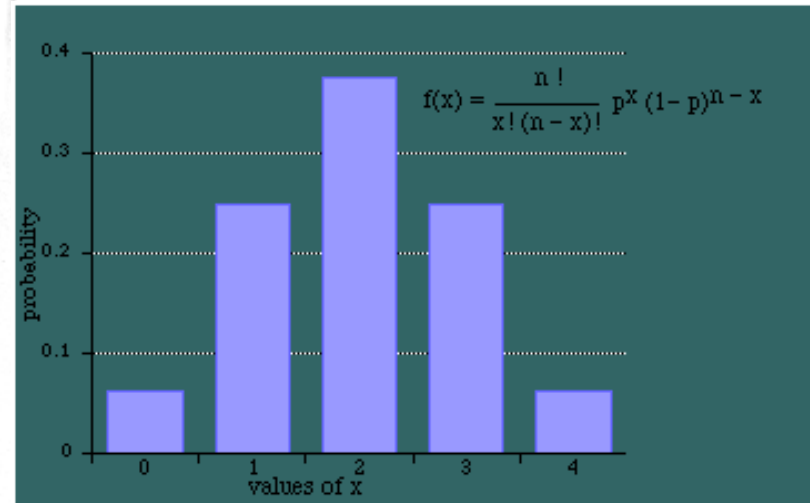
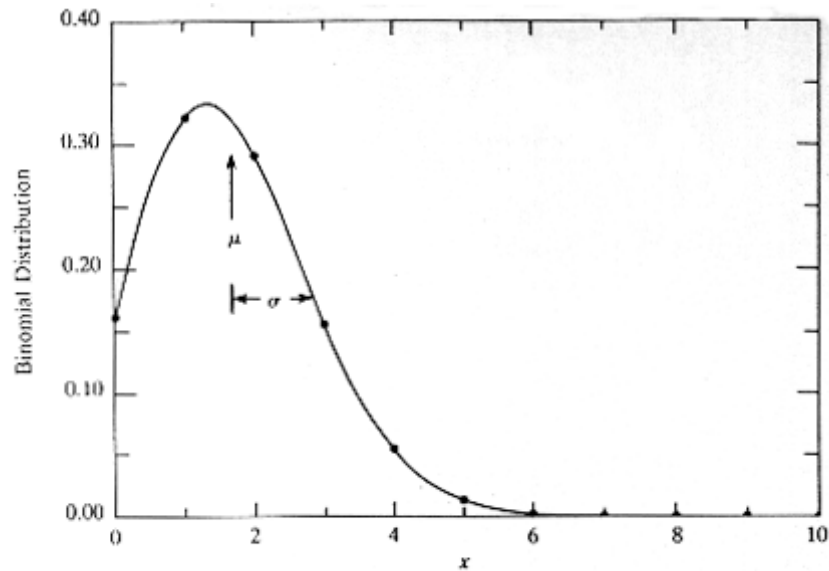
$$P_p(n|N) = \binom{N}{n} p^n q^{N-n}$$

Binomial  
coefficient

20 trials, with  $p = q = 1/2$

Symmetric only if  $p = q$ .

# Ch 10 Binomial Distribution



The **binomial distribution** describes the behavior of a count variable  $X$  if the following conditions apply:

- 1:** The number of observations  $n$  is fixed.
- 2:** Each observation is independent.
- 3:** Each observation represents one of two outcomes ("success" or "failure").
- 4:** The probability of "success"  $p$  is the same for each outcome.

# Binomial Distributions in Practice

- You should really know when to use the Gaussian hypothesis. When the number of attempts/trials is  $> 15$ , you are safe.

Then :

$$\mu_X = np$$
$$\sigma_X^2 = np(1-p)$$

- Then use the one or two sided t-probability distributions to get the probability.
- This is nice also because calculating the factorial is very computationally demanding when  $N > 50$ .

# Example

- What's the probability of getting 27 Heads out of 34 tosses of a coin?

$$B_{27,1/2}(v) = \frac{34!}{27!7} \left(\frac{1}{2}\right)^{27}$$

MICROSOFT EXCEL:

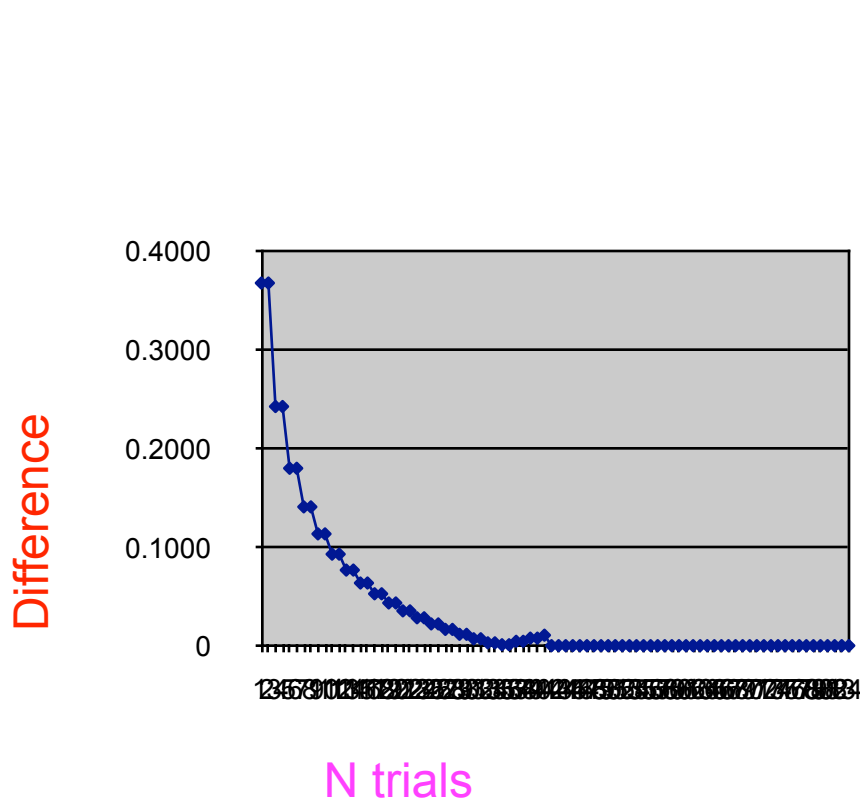
=BINOMDIST(23,36,0.5,FALSE)

$$G_{x=17, \sigma = \sqrt{17(0.5)}}(v) = \frac{1}{2.6\sqrt{2\pi}} \exp\left[-\frac{(27 - \bar{17})^2}{2(2.9)^2}\right]$$

MICROSOFT EXCEL:

=NORMDIST(x,mean,standdev,FALSE)

# How Good is Gauss?



numb success	total N	Binomial (exact)	Normal (Approx)	difference
23	36	0.033626414	0.033159046	0.000467
1	1	0.5	0.131146572	0.368853
2	2	0.25	0.125794409	0.124206
3	3	0.125	0.117355109	0.007645
4	4	0.0625	0.106482669	-0.04398
5	5	0.03125	0.093970625	-0.06272
6	6	0.015625	0.080656908	-0.06503
7	7	0.0078125	0.067332895	-0.05952
8	8	0.00390625	0.054670025	-0.05076
9	9	0.001953125	0.043172532	-0.04122
10	10	0.000976563	0.033159046	-0.03218
11	11	0.000488281	0.024770388	-0.02428
12	12	0.000244141	0.017996989	-0.01775
13	13	0.00012207	0.012717541	-0.0126
14	14	6.10352E-05	0.00874063	-0.00868
12	15	0.013885498	0.043172532	-0.02929
16	16	1.52588E-05	0.003798662	-0.00378
17	17	7.62939E-06	0.002402033	-0.00239
18	18	3.8147E-06	0.001477283	-0.00147